On the Retrogradation of the Plane of Saturn's Ring and of those of his Satellites whose Orbits coincide with that Plane. By Professor J. A. C. Oudemans.

Being occupied—in behalf of the second volume of the fourth edition of Kaiser's *Sterrenhemel*—with a discussion of the elements of the satellites and the ring of *Saturn*, I was struck by the following circumstance.

As is generally known, Bessel occupied himself at three different epochs with a research on the apparent diameter and the position of the ring of *Saturn*, 1° in 1812, *Königsberger Archiv für Naturwissenschaften und Mathematik*, 1812, 1r Band, p. 113; 2° in 1818, *Berl. Jahrbuch*, 1829, p. 175; and 3° in 1830–35, *Astronomische Nachrichten*, vol. xii. p. 153. These three papers* have been reproduced by Engelmann in his edition of Bessel's *Abhandlungen*, vol. i. pp. 110, 319 and 150 respectively.

In his first paper Bessel determined the inclination of the ring, using his measures both of the major and of the minor axis, made at a time when it was widely open, in Schröter's observatory at Lilienthal, by the method of projections. The longitude of the node was found by discussing the observations of disappearance and reappearance of *Saturn's* ring, made by several astronomers from 1714 to 1803.

If there were no retrogradation of the ring, the yearly increase of the node's longitude should necessarily have been put $=50''{\cdot}2$, but Bessel found this increase to be only $40''{\cdot}57$, admitting thus a retrogradation of about $9''{\cdot}6$; he did not distinctly pronounce himself on what manner this retrogradation is found, but from the connection between the different parts of the paper I think we must conclude that, as in his third paper, it was introduced as an unknown quantity into the equations formed by the disappearances and reappearances of the ring.

In his second paper Bessel gives the measures of the inclination of the ring, made in 1818 with very moderate means; in fact, the instrument employed was an equatorial of Dollond, provided with a telescope of 16 inches (focal distance); in the field of the strongest eyepiece there was stretched a single wire, and by moving the instrumental arc of this instrument till this wire was parallel to the major axis of the ring, and by reading the latitude-arc and the noniuses, the inclination of the ring to the vertical was found. It is not surprising that these observa-

* Professor E. S. Holden, comparing his and Professor Asaph Hall's determinations of the inclination of *Saturn's* ring, made at Washington during the years 1877, 1878, and 1879, with Bessel's (*Monthly Notices*, April 1882), overlooks the last and most renowned determination, and cites only that of 1818. This explains the large probable error of a single observation of Bessel's, viz. $24'{\cdot}1$, whereas Hall's is found to be $8'{\cdot}6$, and Holden's $10'{\cdot}3$.

tions (the only one cited by Professor Holden) possess the relatively high probable error of $\pm$ 24′·1.

Bessel became possessed of a heliometer in March 1829, and in his third paper he follows the same method as in his first; 131 measures, made with the heliometer, of the inclination of the major axis of the ring, taken at an epoch when it was very narrow, gave him 67 equations between δn, δi and m, where

δn is the correction of the adopted node, 167° 21′ 0″, for 1833;

δi, the correction of the adopted inclination, 28° 9′ 30″, for 1833; and

m, the annual variation of the node of the ring's plane on that of the planet's orbit;

and the observed disappearances and reappearances of the ring gave between the same unknown quantities 20 equations, out of which only 15 have regard to observations made on two successive days, viz. before and after the phenomenon, and these equations alone were taken into account.

The 67 equations of the first order, of weights varying from 1 to 4, have all small coefficients of δn and m, but a negative coefficient of δi not much differing from unity, so they have been combined according to their weight, and gave the following mean equation:

$$(131 \text{ observations}) \ldots 0 = 65''\cdot2 + 0\cdot0452\,\delta n - 0\cdot9647\,\delta i + 0\cdot25\,m.$$

The fifteen equations of the second order have all nearly the same positive or negative coefficient of δn, viz. from 0·4700 to 0·4710. The coefficients of δi have all their signs contrary to that of δn, and a value of from 0·0657 to 0·0894. Excepting the two last equations given by the observations of the disappearance on April 26 and the reappearance on June 13, 1833 (i.e. *after* the epoch 1833·0), the coefficient of m has the same sign as that of δn, but it diminishes from 53·28 (1714, October 15) to almost naught. (N.B. 0·47 is the sine of the inclination 28° 9′ 30″ and the coefficient of m is = the coefficient of δn multiplied by the number of years before the epoch.)

Believing (probably after the result of his former investigation) that the value of m was nearly = 10″, Bessel introduced the new unknown quantity $m' = m - 10$, but afterwards he returned to m, and accordingly we will employ this letter only. Treating the fifteen equations of the second order by the method of least squares, Bessel found:

$$0 = -151\cdot59 + 3\cdot3205\,\delta n - 0\cdot5486\,\delta i + 167\cdot949\,m$$

$$0 = -23577 + 167\cdot949\,\delta n - 27\cdot317\,\delta i + 12635\cdot3\,m$$

Combining these equations with the one given above,* Bessel found:

* This equation is repeated in Bessel's paper, in the *Astronomische Nachrichten*, in column 167, but with the wrong sign of the term containing δi ($+0\cdot9647\,\delta i$, instead of $-$), and this error is repeated in the reprint in Engelmann's edition of the *Memoirs of Bessel.*

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$$\left.\begin{aligned} \delta i &= +\ 63'',1 \\ \delta n &= -137'',9 \\ m &= +\ \ 3'',848 \end{aligned}\right\}*$$

and as

$$n = 167^\circ\ 21'\ \ 0'' + \delta n + (50''{\cdot}125 - 0{\cdot}95206\, m)\,(\mathrm{T} - 1833)$$
$$i = \ 28^\circ\ \ 9'\ 30'' + \delta i - (\ 0''{\cdot}487 - 0{\cdot}03553\, m)\,(\mathrm{T} - 1833)$$

he gives his result in the following shape:

$$n = 166^\circ\ 53'\ \ 8''{\cdot}9 + 46''{\cdot}462\,(\mathrm{T} - 1800)$$
$$i = \ 28^\circ\ 10'\ 44''{\cdot}7 - \ 0''{\cdot}350\,(\mathrm{T} - 1800)$$

in which they are generally adopted. The astronomical almanacs start from these formulæ to calculate annually the elements of *Saturn's* ring, &c. Even those astronomers who make researches upon the orbits of the satellites, so far as they assume these orbits to coincide with the plane of the ring, accept them as established facts.

It appears to me that the result, published by Bessel, is not so accurate and trustworthy as it seems to be. The coefficient $46''{\cdot}462$ is given to a thousandth of a second, but I have a strong doubt whether even the units are to be depended upon.†

Bessel has not added any discussion of the probable errors of his results, though this was not very troublesome. The equations of the first order stand apart, they can only serve to determine δn; after having substituted into them the values of the unknown quantities, there remain sixty-seven errors. Squaring them and multiplying each square by the weight, the result is (m being the mean error):—

$$(67 - 1)\,\mathrm{m}^2 = 22458$$
$$\mathrm{m}^2 = \ \ 340{\cdot}3$$
$$\mathrm{m} = \pm\ \ 18'{\cdot}45 \quad (0{\cdot}6745\,\mathrm{m} = \pm\,12'{\cdot}44).$$

The equations of the second order that served to determine δn and m were only fifteen in number. Since the laws of the oscillations of the rings are unknown, we are obliged, if we wish to form a judgment on the uncertainty of the result, to consider the remaining differences as errors of observation. So we obtain for these equations:

$$(15\ \ 2)\,\mathrm{m}^2 = \ 253{\cdot}07$$
$$\mathrm{m}^2 = \ \ 19{\cdot}47$$
$$\mathrm{m} = \pm\ 4'{\cdot}41$$

* This solution is not exact. The true values are given below.

† It seems to me to be an abuse of the method of least squares to publish a result with more than one figure than can be accounted for, and especially, as was here the case, if the mean or probable error is not added. *One* figure may be admitted, to indicate if the last figure but one be full or not full. Encke's famous value *in four decimals!* of the Sun's parallax, $8''{\cdot}5776$, and still more his second improved one IN FIVE DECIMALS!! $8''{\cdot}57116$, *whereas the first decimal was wrong,* ought to be a warning example to every astronomical calculator.

Adding now the mean and probable errors to the values of the unknowns as we found them, we have:

	Equation of the order.	Weight.	Value found.		Mean Error.	Probable Error.
δi	1st	131	$+62''.1$	$=+1'.0$	$\pm 1'.6$	$\pm 1'.1$
δn	2nd	1.08	$-138''.1$	$=-2'.3$	$\pm 4'.2$	$\pm 2'.9$
m	2nd	4139.6	$+3''.836$		$\pm 4''.1$	$\pm 2''.8$

Thus we see that the value found by Bessel for m, i.e. the yearly retrogradation of *Saturn's* ring, with respect to his orbit, though larger than its probable error, is at all events smaller than its mean error.

Let us consider now the theoretical value of this retrogradation. Laplace has already, in his most splendid *Mécanique Céleste*, treated the subject, and though we are now able to substitute better values for the constants he employs, his results are still of force. In the Fifth Book he proves that those satellites whose orbits lie in the plane of the planet's equator (i.e. all except *Iapetus*) are kept in that plane by the rotating and elliptic planet, and as the ring may be considered as an aggregation of satellites it is in the same case. And he closes that book by the remark, that *Saturn's* equator, in its very slow motion, caused by the action of the Sun and *Iapetus*, carries with it the planes of the rings and of the satellites.

In § 37 of the Eighth Book he finds for the solar precession of Saturn

$$0.878 \text{ centesimal seconds} = 0''.2845,$$

and for that due to the action of *Iapetus*

$$6195 \text{ centesimal seconds} \times L = 2007''.2\, L,$$

L being the mass of this satellite, that of *Saturn* being taken as unity. By the fact that there has not been found any inclination of *Titan's* orbit to the ring's plane, nor the existence of a fixed plane, making an angle with the ring's plane, and with which *Titan's* orbit makes a constant angle, whereas the line of the nodes has a retrograde motion, Laplace proves that the mass of *Iapetus* is very small—smaller, indeed, than $\frac{1}{200}$th of that of *Saturn*. We may go farther, thanks to the beautiful photometric investigations of Professor Pickering. Taking as an approximation, according to Table LII. on p. 269 of the Ninth Volume of the "Annals of the Harvard College Observatory," the equivalent *mean* diameter of *Iapetus* $= 0.0069 \times$ that of *Saturn*, and the ellipticity of that planet $= \frac{1}{9.2}$, we have:

$$\text{Volume of } Iapetus = \frac{9.2}{8.2} \cdot 0.0069^3 \text{ volume of } Saturn$$
$$= 0.00000036837 \text{ volume of } Saturn$$
$$= \frac{1}{2713000} \text{ volume of } Saturn.$$

Knowing nothing about the density of this satellite, the safest way is to take it equal to that of the planet, then the satellite's mass is expressed by the same fraction, that of *Saturn* being taken as unity. And so we find for the precession of *Saturn's* equator due to *Iapetus* the approximate value of $0''{\cdot}00074$.

I have myself made the calculation of these precessions in the following way:—Supposing the densities of the ellipsoidic layers in the Earth and in *Saturn* to follow the same law, and putting

	For the Earth.	For *Saturn*.
The solar precession	ψ_e	ψ_s
The time of the rotation on the axis	d_e	d_s
The excentricity of the meridian	e_e	e_s
The inclination of the equator upon the plane of the orbit	i_e	i_s
The mean distance from the Sun	a_e	a_s
The angle of excentricity of the orbit	ϕ_e	ϕ_s

we have:

$$\psi_e : \psi_s = \frac{d_e e_e^2 \cos i_e}{a_e^3 \cos^3 \phi_e} : \frac{d_s e_s^2 \cos i_s}{a_s^3 \cos^3 \phi_s}$$

Taking

$$d_e = 1436^{\mathrm{m}} \qquad d_s = 615^{\mathrm{m}}$$

$$e_e^2 = \frac{1}{145} \qquad e_s^2 = \frac{9{\cdot}2^2 - 8{\cdot}2^2}{9{\cdot}2^2} = \frac{17{\cdot}4}{84{\cdot}64}$$

$$i_e = 23^\circ\ 27'{\cdot}3 \qquad i_s = 26^\circ\ 49'{\cdot}5$$

$$a_e = 1 \qquad a_s = 9{\cdot}53885$$

$$\phi_e = 0^\circ\ 57'{\cdot}65 \qquad \phi_s = 3^\circ\ 12'{\cdot}9$$

$$\psi_e = 15''{\cdot}82^*$$

we find:

$$\psi_s = 0''{\cdot}2273.$$

To compare further the action of the Sun ψ_s to that of *Iapetus* ψ_j, we have:

$$\psi_s : \psi_j = \frac{\text{Mass of Sun} \times \cos i_s}{a_s^3 \cos^3 \phi_s} : \frac{\text{Mass of } \textit{Iapetus} \times \cos \mathrm{I}}{a_j^3 \cos^3 \phi_j}.$$

* There is an uncertainty of a little more than $0''{\cdot}1$ in this quantity. Adopting the lunisolar precession $= 50''{\cdot}35$, and the Moon's mass $= \frac{1}{81{\cdot}5}$, I get $15''{\cdot}93$. Nyrén takes the lunisolar precession for $1850 = 50{\cdot}3164$, which value he deduced from the catalogues of Weisse and of Schjellerup (*Bestimmung der Nutation der Erdachse,* St. Petersburg, 1872, p. 54). From his nutation-constant I find the Moon's mass $= \frac{1}{80{\cdot}5}$ and the solar precession $15''{\cdot}79$; but if I take the lunisolar precession to be $= 50''{\cdot}35$, more conform to Ludwig Struve's last determination, we get $15''{\cdot}82$ for the solar only, as we have adopted above.

I signifying the inclination of the orbit of *Iapetus* upon the equator of *Saturn* = 13° 28′ :

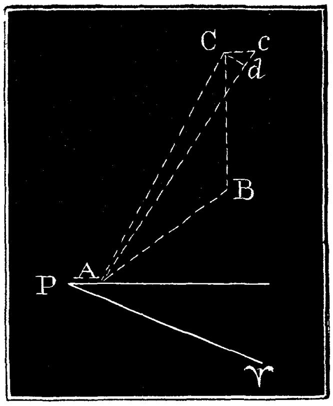
a_j, the semi-axis major of *Iapetus's* orbit $= a_s \sin 515''\!\cdot\!5$

ϕ_j, the angle of excentricity of *Iapetus's* orbit = 1° 35′·6.

The result is :

$$\psi_j = 0''\!\cdot\!001665$$

But this regards a turning of the pole of *Saturn's* equator around the pole of *Iapetus's* orbit, whereas we must find the turning of the same pole around the pole of *Saturn's* orbit.



Let, in the accompanying diagram,

A be the pole of *Saturn's* orbit,
B that of *Iapetus's* orbit,
C that of *Saturn's* equator,
P that of the ecliptic,
♈ the vernal equinox.

We have for 1850 :

$$\begin{aligned} \Upsilon\,PA &= 22^\circ\ 21' \\ PA &= \ \ 2\ \ 30 \\ \Upsilon\,PB &= 52\ \ 28 \text{ and } APB = 30^\circ\ 7' \\ PB &= 18\ \ 39 \\ \Upsilon\,PC &= 77\ \ 36 \text{ and } APC = 55^\circ\ 15' \\ PC &= 28\ \ 10 \end{aligned}$$

And herewith we find :

$$AB = 16^\circ\ 32' \qquad AC = 26^\circ\ 48'\!\cdot\!8$$
$$PAB = 145\ \ 40\cdot5 \qquad PAC = 120\ \ 42\cdot4$$
$$\therefore\ CAB = 24^\circ\ 58'\!\cdot\!1$$

In the spherical triangle ABC, AB, AC and the angle BAC being known, we find :

$$BC = 13^\circ\ 36' \qquad \text{Angle } C = 30^\circ\ 43'$$

The turning around B takes place in the directions C*c*, perpendicular to BC, but we want to know the turning around A, so letting fall the perpendicular C*d* we have :

$$C\,c = \psi_j \sin B\,C$$

$$C\,d = C\,c \cos C$$

$$C\,A\,d = \frac{C\,d}{\sin A\,C} = \frac{\sin B\,C \cos C}{\sin A\,C} \cdot \psi_j$$

$$= 0''\cdot 000746$$

in full accordance with Laplace. Adding the solar precession $0''\cdot 2273$, the *Iapeti-solar* precession becomes $0''\cdot 2280$.

There remains now to consider the action of the ring, retrograding itself by the Sun's attraction, on the planet.

The ring may be considered as an aggregation of satellites. Taking the mean semi-diameter of the ring at *Saturn's* mean distance $= 16''\cdot 75$, we find, by comparison with the satellites, the time of rotation $= 0\cdot 4647$ days $= t$.

The same element of *Saturn* is 10759 days $= T$.

A fair approximation of the time of retrogradation of the nodes of such a satellite is (Möbius, *Mechanik des Himmels*, p. 223):

$$\frac{4}{3} \cdot \frac{T^2}{t} \text{ days} = 322200000 \text{ days}$$

$$= \quad 909500 \text{ years.}$$

So that the yearly retrogradation of the ring's node, if the planet were not elliptical, should be

$$1''\cdot 425.$$

If the ring were solidly attached to the planet it would draw it along, and the result would be a retrogradation between $0''\cdot 228$ and $1''\cdot 425$, easily to be determined, taking into account the proportion of the different moments of inertia.

Calculating only the moments of inertia with regard to the rotatory axis, we find

Mom. of inertia of planet $=$ mass of planet $\times \frac{2}{5} a^2$

,, ,, of the ring $=$ mass of the ring $\times \frac{1}{2} (r^2 + r'^2)$.

Taking

$r' = 19''\cdot 75$, the outer radius of the ring,

$r = 13''\cdot 75$, the inner radius of the ring,

$a = 8''\cdot 65$, the radius of *Saturn's* equator

(the final values, which we arrived at after careful discussion) we find, if the mass of the ring $= \frac{1}{\mu} \times$ the mass of *Saturn* :

$$\frac{\text{Moment of inertia of ring}}{\text{Moment of inertia of } Saturn} = \frac{9\cdot 688}{\mu}.$$

The fraction $\frac{1}{\mu}$ is yet very uncertain. Bessel, disregarding the ellipticity of the planet, made it $\frac{1}{118}$, but was, of course, aware that this value was too great. Tisserand, to explain a very vague expression of Captain Jacob: "The apsides (of *Mimas*) seem to have made nearly a semi-revolution in the course of a year," and having found by theory that by the planet's ellipticity alone the line of *Mimas's* apsides has a yearly motion of 349°, supposes that motion to have been (between January 1857 and January 1858, the epochs of Jacob's observations) $1\frac{1}{2}$ revolution, and in that supposition

$$\text{“ On trouverait } \frac{\mathrm{m}}{\mathrm{M}} = \frac{1}{620}\text{;”}$$

but, considering that the observations of Professor Asaph Hall at Washington, made with much more powerful means than Jacob's, do not reveal an ellipticity of the orbit of *Mimas*, I believe this value cannot be properly named a result, especially as Jacob himself acknowledges (*Memoirs Royal Astronomical Society*, vol. xxviii. p. 69) "The observations registered of *Enceladus* and *Mimas* are in general little better than *estimates* of the direction," and as an error in the position-angle is enlarged when reduced to an error in the satellite's kronocentric longitude, the observations of these satellites being taken only in the elongations.

The latest valuation of the ring's mass is that of Hermann Struve,* who, consulting the motions of the apsides, both the direct one of *Titan* and the retrograde one of *Hyperion*, and adopting the best available measures of *Saturn's* equatorial and polar diameters, treats the problem as leading to two equations with two unknowns, viz. the mass of *Titan* (which is found $\frac{1}{4678}$), and the mass of the ring, for which is found the *superior limit* of $\frac{1}{314}$.

Adopting this value, we find for the superior limit of the moments of inertia of the ring and the planet about $\frac{1}{32 \cdot 4}$, and so the common retrogradation of the ring and the planet could not rise higher than

$$0''\cdot 228 + \frac{1}{33 \cdot 4} \times 1''\cdot 197 = 0''\cdot 264.$$

Now the ring is not solidly attached to the planet, but as the spheroidic planet keeps the ring in the plane of the equator, the

* In his beautiful memoir: *Beobachtungen der Saturnstrabanten*, 1e Abtheilung, 'Beobactungen am 15-zölligen Refractor.' Supplément I. aux *Observations de Poulkova*, St. Pétersbourg, 1888.

effect is likely to be not much less, and so I believe that, taking a round number,

$$0''\cdot 25,$$

is a fair approximation to the precessional retrogradation of the ring and planet together.

I return to Bessel's solution. Bessel gave his results in a form as if the mean epoch of the observations were 1800, but, on the contrary, it was 1833·0 for his own determinations of the inclination, and 1780 for the determination of the node. The consequence is that if we do not adopt the retrogradation of the node found by him ($3''\cdot 848$) we shall find another longitude of it for 1800, but the same for 1780 and this one with the maximum of weight. Indeed, we find for 1780

Weight.	Mean error of δn.	Probable error.
3·295	$\pm 2'\cdot 4$	$\pm 1'\cdot 6$

The yearly increase of the longitude of the ring's node with the ecliptic becomes now:

$$50''\cdot 125 - 0\cdot 95206\, m = 49''\cdot 89$$

And so there follows from our considerations that the results of Bessel's investigation ought to be written in the following way:

	Probable Error.	Yearly Variation.
For 1780 : $n = 166^\circ\ 37'\cdot 7$	$\pm 1'\cdot 6$	$+49''\cdot 89 = +0'\cdot 8315$
For 1833 : $i = 28\ \ 10\cdot 7$	$\pm 1\cdot 1$	$-\ 0\cdot 48 = -0\cdot 0080$

or in other words:

$$n = 166^\circ\ 37'\cdot 7 + 0'\cdot 8315\ (T - 1780),$$
$$i = 28^\circ\ 10'\cdot 7 - 0'\cdot 0080\ (T - 1833).$$

The difference between my result and Bessel's is not so very small; it is $3''\cdot 6$ per annum. Bessel's mean year (not given by him) was 1780; consequently we have now the effect of 109 years, and $109 \times 3''\cdot 6 = 392'' = 6'\cdot 5$. But as *Saturn* is at a distance of nearly 10 unities, a change of direction of the node-line of $6'\cdot 5$ corresponds in the neighbourhood of the Sun to about 1° of the circumference of the Earth's orbit. Thus the difference in a disappearance or reappearance when the Earth passes the plane will be *one day at least*, but generally more; and when the *Sun* passes the plane, it is at least three days, owing to the so much slower motion of *Saturn*.

Since 1835 we have had three epochs of disappearances and reappearances of the ring, viz. 1848–49, 1861–62, and 1878, and the next to come is 1891. In the following table I have collected the times of disappearance and reappearance according to Bessel, and I have added the correction, necessary if our formula be adopted.

		Authority.	Epoch according to Bessel.		Correction.		Resulting epoch.		
				h	d	h		h	
1848	Disappearance	B. J.	April 21	21·4	−0	14·8	April 21	6·6	♁
	Reappearance	,,	Sept. 3	3·3	+1	23·0	Sept. 5	2·3	☉
	Disappearance	,,	,, 12	12·8	+0	22·2	,, 13	11·0	♁
1849	Reappearance	,,	Jan. 19	8·3	−0	15·4	Jan. 18	16·9	♁
1861	Disappearance	,,	Nov. 23	1·05	+1	7·6	Nov. 24	8·65	♁
1862	Reappearance	,,	Jan. 31	19·5	−1	9·6	Jan. 30	9·9	♁
	Disappearance	,,	May 18	2·8	+2	7·1	May 20	9·9	☉
	Reappearance	,,	Aug. 12	20·2	+0	16·4	Aug. 13	12·6	♁
1878	Disappearance	N. A.*	Feb. 6	11	+2	20	Feb. 9	7	☉
	Reappearance	,,	Mar. 1	4	−0	18·6	Mar. 0	9·4	♁
1891	Disappearance	,,	Sept. 22	18·9	+0	21·0	Sept. 23	15·9	♁
	Reappearance	,,	Oct. 30	10·9	+3	6·0	Nov. 2	16·9	☉

It is not yet possible to decide if the epochs of the last column satisfy better than Bessel's. I found the following observations of the disappearances and reappearances:

1848. First disappearance. (A.R. ♄ − A.R. ☉ = $21^{h}\cdot5$.) No observations found.

First reappearance. (♄ − ☉ = $12^{h}\cdot8$), Bond. Between August 31, 1878, and September 3·80, G.M.T. (*Monthly Notices*, vol. x. p. 17).

September 3, Schwabe: "no trace" (*Astronomische Nachrichten*, 665).

September 4, $9^{h}\cdot5$, Schwabe: sees the ring (6 f. telescope).

Julius Schmidt, at Bonn, sees the ring September 3 (heliometer and 5 f. telescope). (*Astronomische Nachrichten*, 648).

Busch, at Königsberg, September 5 (4-inch Plössl.), after cloudy weather (*Astronomische Nachrichten*, 658).

Galle, at Berlin, September 5, "zuerst im Refractor" (*Astronomische Nachrichten*, 647).

Dawes, September 1. Power 163. *No trace of the ring* (*Monthly Notices*, vol. x. p. 47).

September 2, 11^{h} G.M.T. Power 163. *The ring is visible* as an excessively narrow line *of nearly the same colour as the planet*.

* In the *Berliner Jahrbuch* of 1878, the elements of the ring of *Saturn* were omitted.

Second disappearance (♄ − ☉ = $12^{h}\cdot1$), Bond: between September 12·80 and 13·62 G.M.T. (l.c.) Schmidt sees the ring September 11, but saw it not September 14 at 8^{h} (*Astronomische Nachrichten*, 850). Busch: a trace September 13, no ring September 15 (*Astronomische Nachrichten*, 658). Schwabe: September 12 only the western ansa, the 13th no trace (*Astronomische Nachrichten*, 665).

Second reappearance 1849, January (♄ − ☉ = $3^{h}\cdot4$). Bond: between January 18·47 and 19·43 (l.c.). Schwabe: January 19 not visible, January 21 a thin line (*Astronomische Nachrichten*, 667). Schmidt: January 18 not, 21 well visible (*Astronomische Nachrichten*, 650).

1861. First disappearance (♄ − ☉ = $19^{h}\cdot6$). Secchi: the 23rd of November only a feeble line of light ("*un* faible trait de lumière"). (*Astronomische Nachrichten*, 1341.)

1862. First reappearance (♄ − ☉ = $14^{h}\cdot6$). Schwabe sees the ring (after long cloudy weather ? ?) for the first time 1862, February 7 (*Astronomische Nachrichten*, 1384).

Second disappearance (♄ − ☉ = $7^{h}\cdot5$). Dawes, May 17: Scarcely; no shadow of ring! (*Monthly Notices*, vol. xxii. p. 297.) Huggins saw the ring May 17, 18, and 19, and even 20, when the ansæ appeared "of a beautiful dark colour, scarcely distinguishable from the dark blue of the sky, which contrasted strongly with the yellow light of the ball."

Carpenter at Greenwich and O. Struve (at Pulkowa?) saw the ring very well May 19, but also May 20, 25, and June 1; so I suppose it was the border of the ring that they saw, like also Huggins on May 20.

1878. ♄ − ☉ = $1^{h}\cdot8$ at disappearance.
= $0^{h}\cdot9$ at reappearance.
No observations found.

1891. ♄ − ☉ = $23^{h}\cdot5$ at disappearance.
= $21^{h}\cdot4$ at reappearance.

We see that in 1891 it will be impossible to observe the disappearance of the ring, but there is a chance left for the reappearance; the observations, however, must then be made before sunrise.

Utrecht: 1888, *December* 12.

Photographs of the Nebulæ M 31, *h* 44, *and h* 51 *Andromedæ, and M* 27 *Vulpeculæ.* By Isaac Roberts.

Nebulæ M 31, *h* 44, *and h* 51 *Andromedæ.*

The photograph* which accompanies these notes was taken on October 1 last, and it throws a very different light to that hitherto seen by astronomers upon the constitution of the great nebula, and we shall not exaggerate if we assert that it is now for the first time seen in an intelligible form.

No verbal description can add much to the information which the eye at a glance sees on the photograph, and those who accept the nebular hypothesis will be tempted to appeal to the constitution of this nebula for confirmation, if not for demonstration, of the hypothesis. Here we (apparently) see a new solar system in process of condensation from a nebula—the central sun is now seen in the midst of nebulous matter which in time will be either absorbed or further separated into rings. The farthest boundaries of the nebula have already separated into rings more or less symmetrical with the nucleus, and present a general resemblance to the rings of *Saturn*.

The two nebulæ *h* 44 and *h* 51 seem as though they were already undergoing their transformation into planets. But I must refrain from further running riot with the imagination and draw attention to some points shown upon the photograph, premising that we have now the means of keeping a strict watch upon many of the structural details of the nebula which in time may enable astronomers to arrive at a demonstration of the nebular hypothesis.

It will be observed that the nebula *h* 44 is shown on Bond's drawing with its major axis forming an angle of about 45° with a line joining its centre with the centre of *h* 51, whereas the photograph shows the axis to be pointed much more directly towards that nebula—the angle being less than 20°.

The difference is so obvious that we may reasonably suspect that it is not wholly due to error in charting but is indicative of a change in the direction of the axis since the year 1847, and some confirmation of this supposition is also given by comparison of the distances between the centres of the two nebulæ and the nucleus of the great nebula.

On Bond's chart the distance ratio of *h* 51 and *h* 44 with the nucleus is as 32 to 54, but on the photograph it is as 36 to 54, thus showing a large discrepancy.

M 27 *Vulpeculæ.*

The accompanying photograph of this nebula was taken on October 3 last, and shows more of the structure than the one with shorter exposure which was presented to the Society last year. It would be difficult to recognise in the photograph any resemblance to a dumb-bell, and only in general outline does it resemble the best drawings published of it.

* The photographs are deposited in the Library.